SSM
Scuola Superiore Meridionale

MERC PhD Project Proposal 2022/2023

Title of the research project:

Stochastic control of dynamical systems with constrained state-dynamics

Keywords (up to five)

Stochastic differential equations, stochastic control, partial differential equations

Supervisors (at least two from two different areas):

Supervisor 1 (name, contact details, homepage, area of expertise)
Tiziano De Angelis
https://sites.google.com/site/tizianodeangelis

Supervisor 2 (name, contact details, homepage, area of expertise)

Project description (max 5000 characters)

Please include a description of the work to be carried out. State of the art, key research questions and project objectives, workplan and the methodological and application aspects of the project.

In recent ongoing work with Erik Ekstrom (Uppsala University) we obtained probabilistic closed-form solutions *for stochastic control problems in which the controller must ensure that the stochastic dynamics is bound to evolve in a pre-specified* admissible domain *while at the same time minimizing a certain cost functional. The approach is based almost entirely on methods from the theory of diffusions with some insight from partial differential equations (PDEs) and it shows significant promise in terms of generalisations and real-world applications.*

Problems of this kind are motivated by applications to vehicle routing: imagine controlling a vehicle that needs to travel between two positions in minimal time (or with minimal fuel consumption) and avoid obstacles that it may encounter on its path. The engineering literature on this topic is vast with numerous important contributions both at the theoretical and numerical levels. There are also strong connections to large deviations problems, problems of reachability of sets and stochastic target problems.

In this project the PhD students are going to explore the potential of a fully probabilistic approach to stochastic control problems with state constraints. At the theoretical level the initial objective it to obtain closed-form solutions that should improve on analogue results from the engineering and PDE literature. Even when solutions are not fully explicit, their probabilistic representation will enable Monte Carlo methods for the numerical evaluation of the value function of the optimal control problem, and the associated optimal control. Starting from simple models the PhD students will build general theoretical results and ad-hoc methods capable of describing the dynamics of systems of interest in various applications. The study of the applications is in itself a building block of this project and the students will benefit from the multi-disciplinary environment at MERC in order to find the most promising directions of work.

After having consolidated the understanding of simple models, the students will add new features to the problem, in order to gain realism and broaden the range of applied problems that can be covered. For example, it may be of interest to analyse situations in which the admissible domain, within which the system must evolve, changes dynamically and, possibly, according to some other stochastic process. That corresponds, for example, to the situation of a vehicle travelling on a busy road. Another natural question concerns partial and or progressive observability of the admissible domain. For example, the controller may not know the geometry of the domain at the beginning of the optimization problem. Instead they may be able to learn such geometry depending on which portions of the state space the system visits. Further extensions should be considered by the student, based on the particular applications at hand.

Relevance to the MERC PhD Program (max 2000 characters)

Briefly describe how this project fits within the scope of the MERC PhD program describing its interdisciplinary aspects, relevance in application and beneficiaries.

The project fits within the Complexity area of MERC and also within the Risk area, although perhaps to a lesser extent. For example, routing a vehicle in a rapidly changing environment is a major engineering challenge and one of the possible applications of this project. Adding state constraints to standard stochastic control problems leads to stimulating mathematical questions that will be addressed in this project.

Key references

Day, M., 1980. On a stochastic control problem with exit constraints. Appl. Math. Optim. 6 (2), pp. 181–188.

Fleming, W.H., 1977. Exit probabilities and optimal stochastic control. Appl. Math. Optim. 4 (1), pp. 329–346.

Karatzas, I. and Shreve, S., 2012. Brownian motion and stochastic calculus (Vol. 113). Springer Science & Business Media.

Shah, S., Tanner, H., Pahlajani, C., 2015. Optimal Navigation for Vehicles With Stochastic Dynamics. IEEE Transactions on Control Systems Technology 23, pp. 2003–2009.

Joint supervision arrangements

Describe joint supervision arrangements, e.g. weekly/monthly meetings with one or both supervisors, how will the joint supervision be split etc.

Weekly meetings with T. De Angelis either online or in presence. Monthly meetings with both supervisors.

Location and length of the study period abroad (min 12 months)

Give details of the foreign research institution where the student will be host together with the full name and contacts of the foreign host. Please indicate if the foreign institution has already agreed to host the student and when the student is expected to travel abroad.

A possible host for the period abroad is
Prof. Erik Ekstrom, Uppsala University

I have an ongoing collaborations with Ekstrom on a paper closely related to the one in this project description. Ekstrom is available to host PhD students from MERC.

Any other useful information

E.g. involvement of stakeholders, industrial partners, other research institutions etc, funded research projects related to the proposed activity etc.

Caio Cesar Graciani, who is postdoc at SSM is working on closely related topics, so the PhD student would also benefit from regular interactions with Caio.

Please return this form via email by no later than 24th February 2023